Adaptive Q-law Control for Closed-loop Electric Propulsion Orbit Transfer

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Introduction

- Electrical propulsion systems are increasingly favored for communication satellites due to its high specific impulse
- The orbit transfer operation span hundreds to thousands of revolution over several months due to low thrust
- Trajectory design and execution is highly complex
- Existing approaches to orbit transfer using chemical propulsion utilize ground based planning and tracking of reference commands onboard thus requiring continuous ground intervention
- Open loop planning and execution also builds the maneuver execution error due to inherent uncertainties in the system
- To minimize maneuver execution error, the planning and execution cycle is repeated periodically using the spacecraft states available from ground based orbit determination
- Therefore, closed-loop orbit transfer is beneficial for autonomous spacecraft operations

Existing approaches to orbit transfer

Trajectory Design for low thrust orbit transfer is a mature field

Prominent Approaches

- Direct/Indirect trajectory optimization methods (Kluever 2010, Betts 2000, Graham 2016, Leomanni 2021 etc.)
- Feedback control type heuristics
  - Utilizes sums of squares or thrust blending strategies (Petropoulos 2005, Falck 2014, Varga 2016, Hatten 2012 etc.)
- Combination of Trajectory Optimization with feedback control law (Lee 2005, Shannon 2020)
- Learning based methods (Holt 2021 etc.)

- **Q-law** is one of the most comprehensive frameworks for trajectory planning and has been widely used for **offline trajectory generation**
- **Limitations:** Classical Q-law lacks guaranteed closed-loop stability (Hatten 2012, Holt 2021)
  - Primarily suitable for offline planning but not real-time control
- Holt (2021) introduced RL based framework to optimize the gains that preserves closed-loop stability

Motivation for Current Work

- Real time orbit transfer simplify mission planning
  - Reduced dependence on ground-based corrections
  - Autonomous spacecraft operations
  - Reduced cost of man-power and ground system logistics’
- Communication satellites utilize single processor where multiple tasks are shared within one computer
- This necessitates a computationally lightweight closed-loop control algorithm
  - Retains near-optimal performance achieved through offline planning
  - Robust to state and control perturbations in the real flight

Our Contributions

- We propose a modified Q-law that guarantees closed-loop stability by direct modification of lyapunov terms and derivatives of the classical Q-law
- We the extend the modified Q-law by developing an **evolutionary machine-learning–based gain adaptation** framework
- **Gain adaptation with modified Q-law** forms a **computationally lightweight, closed-loop orbit control algorithm**
- **Enable autonomous, time-optimal orbit transfer** without manual intervention, achieving performance comparable to trajectory-optimization-based methods

Dynamics of Orbital Transfer

The dynamics is expressed in RTN frame using gauss variation equation as,

$$\frac{d\boldsymbol{z}}{dt} = \overbrace{\begin{bmatrix} \frac{p\cos\theta}{eh} & -\frac{(p+r)\sin\theta}{eh} \end{bmatrix}}^{\Phi(\boldsymbol{z})} \boldsymbol{u} + \begin{bmatrix} 0 \\ \frac{h}{r^2} \end{bmatrix}$$

$$\boldsymbol{z} = [a, e, i, \Omega, \omega]^T$$

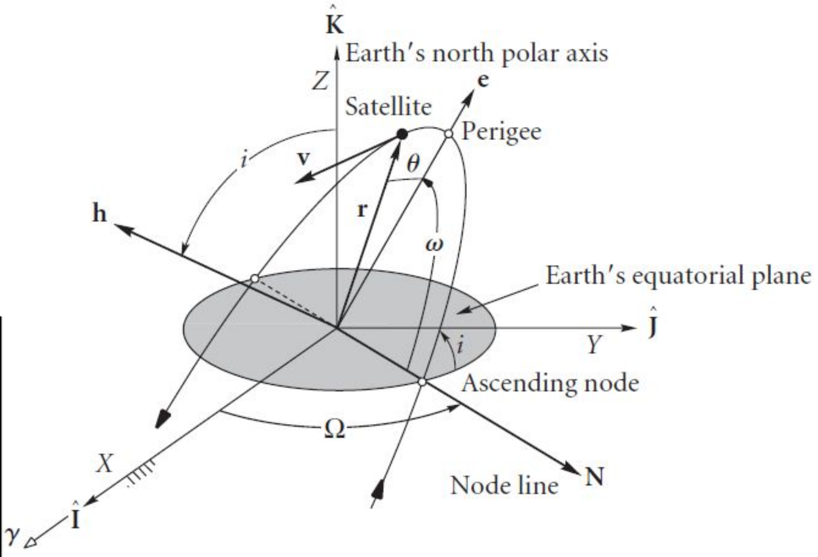
$$\mathbf{u} = [u_r, u_T, u_N]^T$$

Perturbing acceleration vector

$$\Phi(z) = \begin{bmatrix} \frac{2a^2}{h} e\sin\theta & \frac{2a^2}{h}\frac{p}{r} & 0 \\ \frac{p\sin\theta}{h} & \frac{(p+r)\cos\theta + re}{h} & 0 \\ 0 & 0 & \frac{r\cos(\theta+\omega)}{h} \\ 0 & 0 & \frac{r\sin(\theta+\omega)}{h} \\ -\frac{p\cos\theta}{eh} & \frac{(p+r)\sin\theta}{eh} & \frac{r\sin(\theta+\omega)\cos i}{h\sin i} \end{bmatrix}$$

$$\dot{\boldsymbol{z}}_{xx} = \max_{\hat{\boldsymbol{u}},\theta} \dot{\boldsymbol{z}}$$

Maximum rate of change in orbital element over all thrusting direction and true anomaly



a = semi-major axis
e = eccentricity
i = inclination
Ω = Right ascension of ascending node
ω = Argument of perigee
θ = True anomaly

p = semi-latus rectum,
h = specific angular momentum,
r = distance of spacecraft from central body

Orbit Transfer Problem

- In typical electric propulsion system, the total thrust produced by thrusters are fixed
- The control variable is the direction of thrust vector

Orbit transfer problem therefore is a 2-point boundary value problem that computes optimal thrust vector direction given initial and terminal orbit parameters

- True anomaly is considered free parameter in the transfer
- Out of the remaining 5 orbital elements,
  - ω is considered in the design of launch vehicle trajectory as its propellant expensive correction
- This presentation will cover the orbital transfer for remaining orbital elements using spacecraft propulsion

Unified Q-law Formulation (Classical and Modified)

$$V = \sum_{z} W_z K_z(\boldsymbol{z}) d(z, z_T)^2$$

→ State dependent term capturing the orbital dynamics

- V denotes the weighted average of the minimum transfer time for each orbital element

$$\frac{dV}{dt} = \sum_{z} \frac{\partial V}{\partial z}\frac{dz}{dt} = V_z^T \Phi(\boldsymbol{z})\boldsymbol{u}$$

- Thrust vector direction is computed by minimizing derivative of Lyapunov function

$$\hat{\boldsymbol{u}}^* = \arg\min_{\hat{u}} \dot{V} = \frac{\Phi(z)^T V_z}{||\Phi(z)^T V_z||}$$

- Classical Q-law utilizes $K_z(\boldsymbol{z}) = \frac{1}{\dot{z}_{xx}^{\,2}}$ (Classical Q-law also contains scaling terms for semi-major axis)
- Modified Q-law utilizes approximation of $K_z(\boldsymbol{z})$ that utilizes only the dominant dynamics coupling term and preserves closed loop stability

Modified Q-law Formulation

Planar Dynamics (Semi-major Axis and Eccentricity)

$$V_a = \frac{(a-a_T)^2}{\dot{a}_{xx}^{\,2}} = c\,\frac{1}{4a^3}\frac{1-e}{1+e}\,(a-a_T)^2$$

$$\frac{\partial V_a}{\partial a} = \frac{c}{4a^4}\frac{1-e}{1+e}\,(a-a_T)(-a+3a_T)$$

$$\frac{\partial V_a}{\partial e} = -\frac{c}{4a^3}\frac{2}{(1+e)^2}\,(a-a_T)^2 < 0$$

- V_a reaches 0 not only at $a = a_T$ but also asymptotically as a tends to infinity
- $\frac{\partial V_a}{\partial a}$ has multiple equilibrium points
- Therefore, its not a valid Lyapunov function
- $\frac{\partial V_a}{\partial a}$ is unbounded in semi-major axis
- $\frac{\partial V_a}{\partial e} < 0$ implies unbounded growth in eccentricity

We proposed approximated maximum rate of change in semi-major axis as

$$\tilde{\dot{a}}_{xx} = 2f\,\sigma(a < a^*)\sqrt{\frac{a^3(1+e)}{\mu(1-e)}} + \sigma(a \geq a^*)\sqrt{\frac{a^{*3}(1+e)}{\mu(1-e)}},$$

$$a^* = \zeta a_T, \zeta \in (0,3)$$

$$K_a(\mathbf{z}) = \frac{1}{\tilde{\dot{a}}_{xx}^{2}}$$

$$\tilde{V}_a = K_a(a-a_T)^2$$

σ – sigmoid function

Modified Q-law Formulation

Planar Dynamics (Semi-major Axis and Eccentricity)

- Similar issues persists for eccentricity as planar dynamics is coupled
  - Existence of spurious minima as a tends to infinity
  - Unbounded increase in a to reduce V_e $(\frac{\partial V_e}{\partial a} < 0)$
  - Infact, in few simulations, we observed unbounded rise in a far exceeds the target semi-major axis without significantly effecting V_e

We propose the following modifications,

$$\tilde{V}_e = \tilde{K}_e (e - e_T)^2 \quad \tilde{K}_e = \begin{cases} \dfrac{c}{4\min(a, a_T)\,(1 - e^2)} & e \leq 1 - \delta_e \\ \dfrac{c}{4\min(a, a_T)\,(1 - (1 - \delta_e)^2)} & e > 1 - \delta_e \end{cases}$$

Inclination Dynamics

$$V_i = \frac{(i - i_T)^2}{\dot{i}_{xx}^{\,2}} = \frac{c}{a(1 - e^2)} F_i^2 (a - a_T)^2, F_i = \sqrt{1 - e^2 \sin^2 \omega} - e|cos\omega|$$

Consider first order approximation of F_i $\quad \tilde{F}_i \approx \left(1 - \frac{1}{2} e^2 \sin^2 \omega\right) - e\,|cos\omega|$

Following relation holds for the true function and its approximation*,

$$\tilde{\dot{i}}_{xx} \leq \dot{i}_{xx} \Rightarrow \tilde{V}_i \geq V_i$$

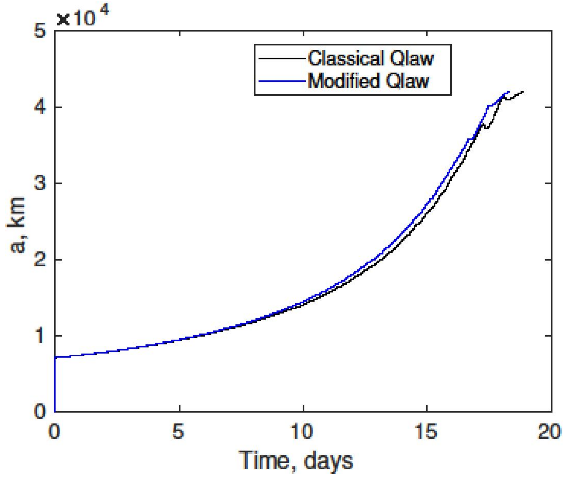
As $\sqrt{V}$ has units of time, the approximation is uniformly an under-approximation of the best case maximum rate of change in inclination

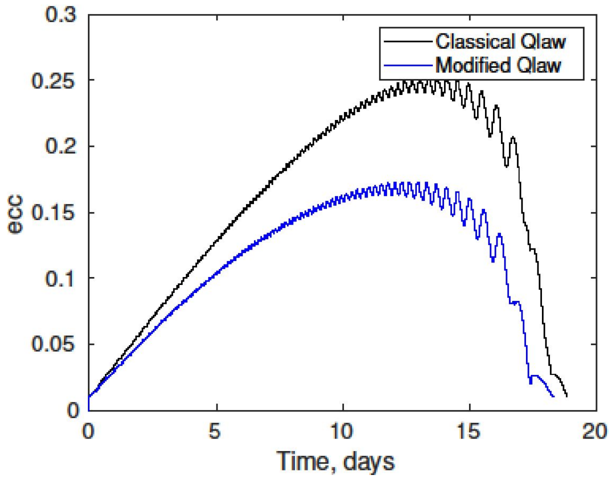
Dependency of lyapunov derivatives w.r.t eccentricity is also forced to 0

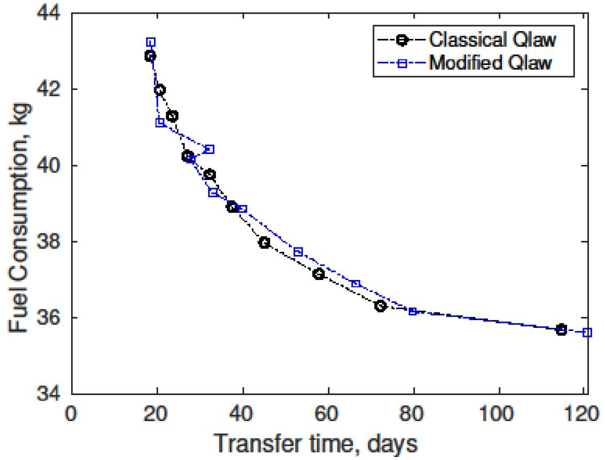
Comparison of Modified Q-law with Classical Q-law

- We showed that modified Q-law with choice of optimal gain (based on gradient free optimization techniques) demonstrates comparable performance with classical Q-law

	Orbit	a(km)	e	inc(deg)		
Co-planar LEO-GEO Transfer	Initial	7000	0.01	0.05	0.0	0.0
	Target	42000	0.01	free	free	free



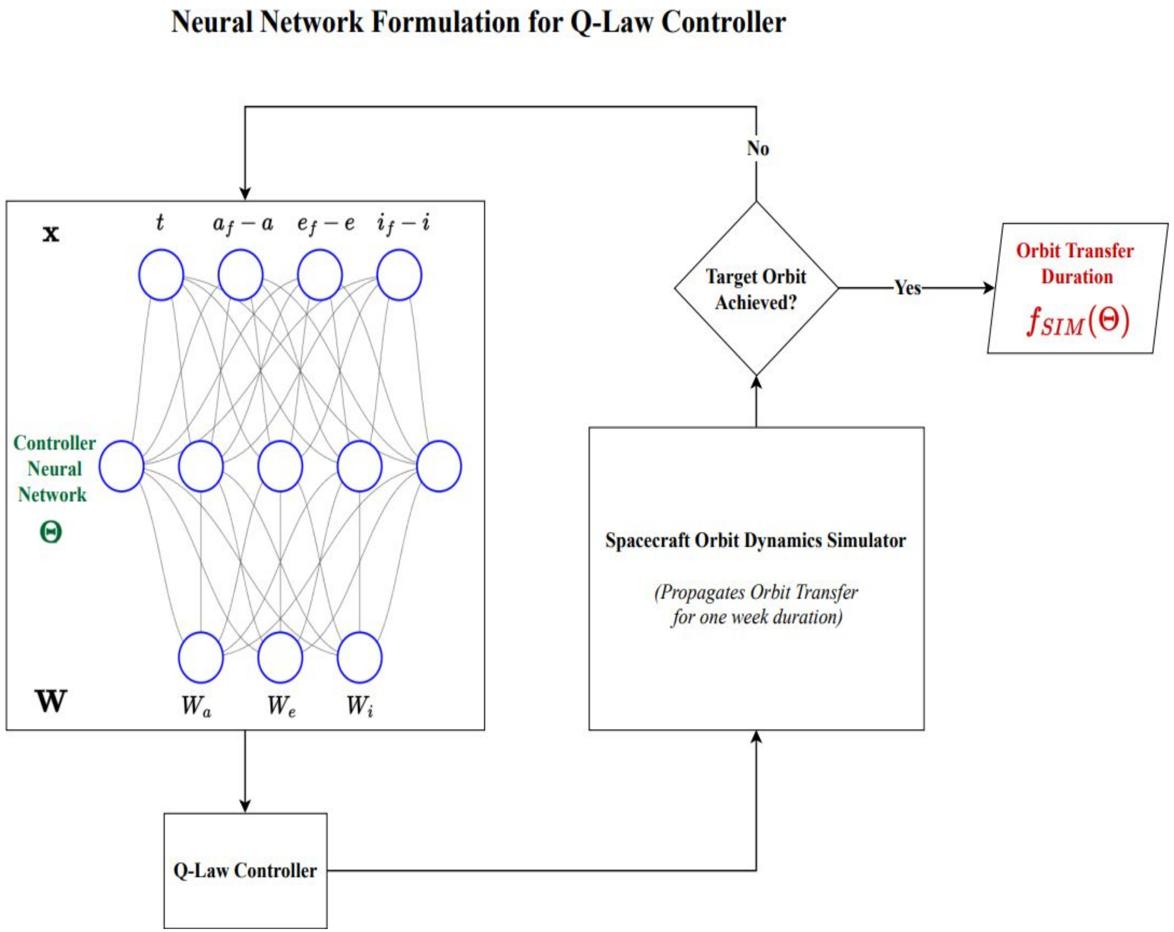


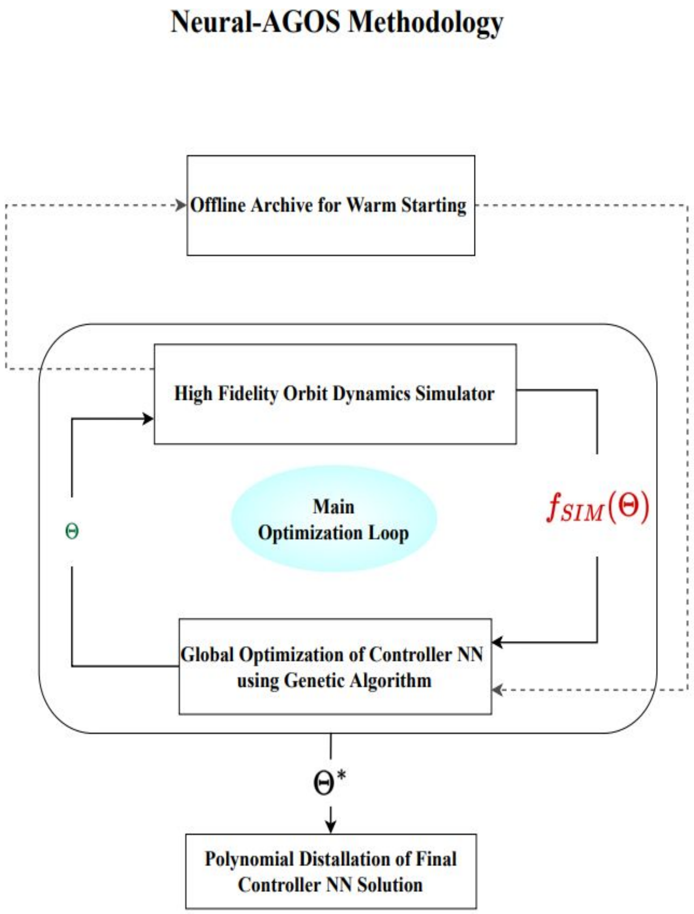


- We ask whether gain adaptation can enhance the performance of the Q-law.
- We propose evolutionary machine learning based simulation based optimization for gain adaptation (applicable to both Classical as well as Modified Q-law)

Adaptive Q-law Formulation via Neural-AGOS

Controller gains are learned as a function of state and time using a neural network parameterization, trained through a two-stage, simulation-based optimization strategy with CMA-ES based evolutionary approach





The acronym Neural-AGOS encapsulates core algorithmic aspects (**Neural** Network Parametrization, **A**rchive-Seeded Warm Start, **G**lobal **O**ptimization of NN, **S**pline Distillation)

Adaptive Q-law Formulation via Neural-AGOS

- **Offline Stage → Archive Seeding for Warm Starting Search:**
  - **Sobol archive + simulation -** Generate N Sobol samples of flattened NN weights $\{\boldsymbol{\Theta}_j^F\}_{j=1}^{N}$; evaluate on the high-fidelity simulator to get costs (transfer time):

$$\{f_{SIM}(\boldsymbol{\Theta}_j)\}_{j=1}^{N}$$

  - **Elite selection -** Keep the best γ-fraction (lowest cost):

$$M = \lfloor \gamma N \rfloor, \qquad \{\boldsymbol{\Theta}_i^F\}_{i=1}^{M}$$

  - **Moments-matched Gaussian for warm starting the CMA-ES (Covariance Matrix Adaptation – Evolutionary Strategy) algorithm:**

$$\boldsymbol{\mu} = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{\Theta}_i^F \quad \boldsymbol{\Sigma} = \alpha^2 \boldsymbol{I} + \frac{1}{M}\sum_{i=1}^{M} (\boldsymbol{\Theta}_i^F - \boldsymbol{\mu})(\boldsymbol{\Theta}_i^F - \boldsymbol{\mu})^T$$

Initialize CMA-ES with $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ to concentrate search in already high-performing regions.

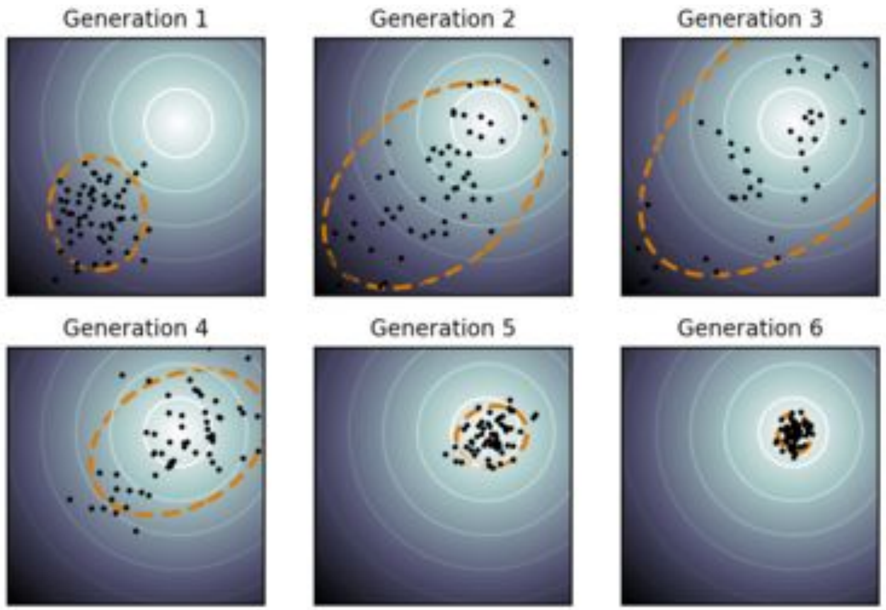
Adaptive Q-law Formulation via Neural-AGOS

 Online Stage Simulator-in-the-loop CMA-ES refinement:

❑ **Sample + evaluate -** Each generation draws candidates from the current search distribution and evaluates directly on high-fidelity sim:

$$\boldsymbol{\Theta}_{\mathbf{k}} \sim \mathcal{N}(\mathbf{m}, \sigma^2 \mathbf{C}) \quad \text{(seeded by } \boldsymbol{\mu}, \boldsymbol{\Sigma}), \qquad f_{SIM}(\boldsymbol{\Theta}_{\mathbf{k}})$$

- ❑ Recombine elites (top-ranked solutions in the population) to update mean.
- ❑ Adapt covariance + step-size (evolution paths): Update $\mathbf{C}$ and σ using cumulative evolution paths to learn correlations in successful steps (natural-gradient-like adaptation), accelerating derivative-free convergence on nonconvex dynamics.



➢ **Distillation (deployable onboard policy):**

- ❑ Compress the converged NN controller into a low-order spline/polynomial gain schedule with a few coefficients (lasso regression) for real-time execution $g(t) \approx \sum_{p=0}^{P} a_p B_p(t)$
- ❑ Enables fast onboard guidance while retaining the optimized time-varying behavior and robustness to disturbances/model mismatch.

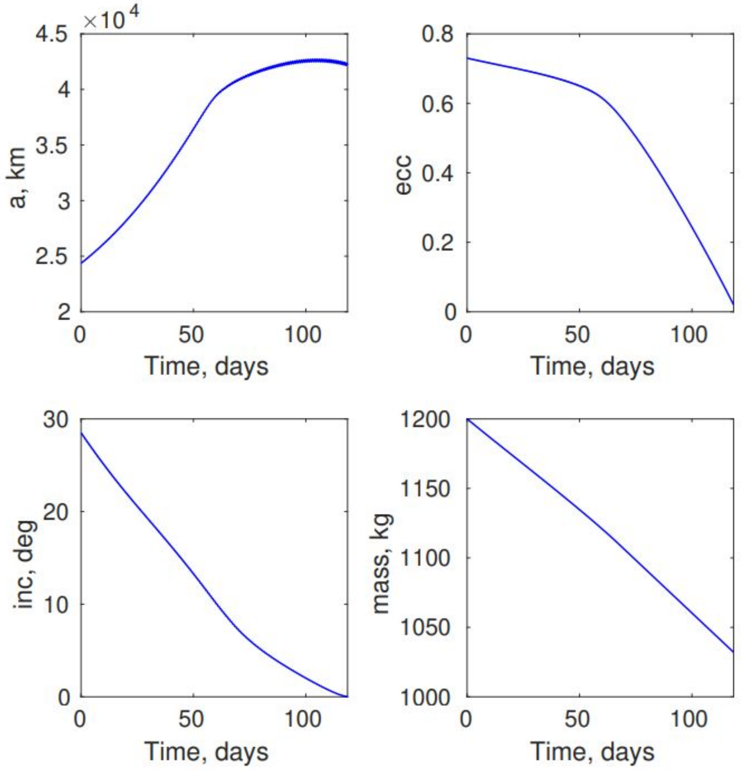
Simulation Results

- Standard GTO-GEO and LEO-GEO transfers are simulated using high fidelity in-house simulation package that models
  - Keplerian dynamics along with perturbations such as J2, J4, solar radiation pressure, sun and eclipse model
  - Electric propulsion thruster functional characteristics such as uncertainty in thrust magnitude, thrust vector misalignment, CG uncertainties etc.

Case	Orbit	a(km)	e	inc(deg)		
GTO-GEO Transfer	Initial	24363.9	0.73	28.5	0.0	0.0
	Target	42164	0.01	0.01	free	free
LEO-GEO Transfer	Initial	6927	0.01	28.5	0.0	0.0
	Target	42164	0.01	0.05	free	free

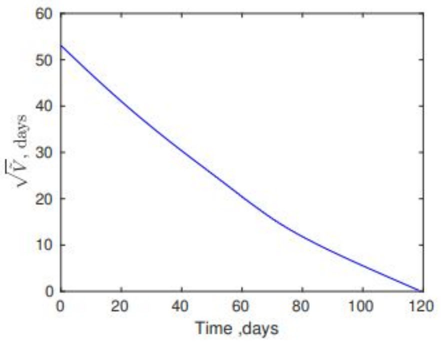
Parameter	GTO-GEO	LEO-GEO
Mass, kg	1200	1000
Thrust, N	0.312	1.445
ISp, s	1800	1850

GTO-GEO Transfer – Time Optimal Transfer

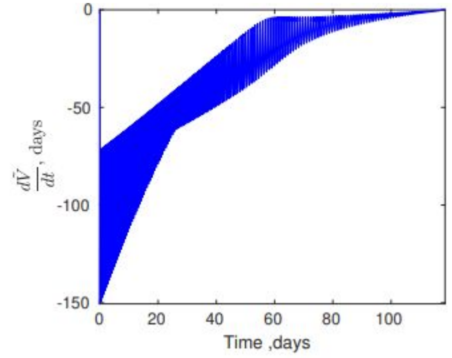


(a) Spacecraft state evolution

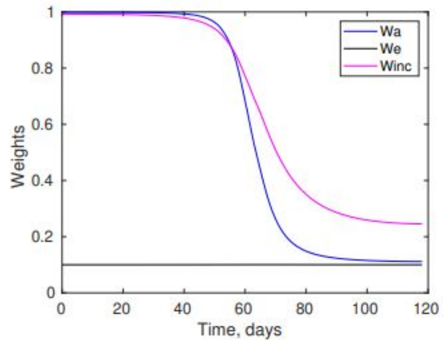
Method	Transfer Time, Days
Literature Best Results [Shannon 2020, Leomanni 2021, Graham 2016]	118-122 days
Adaptive Q-law	118.5 days



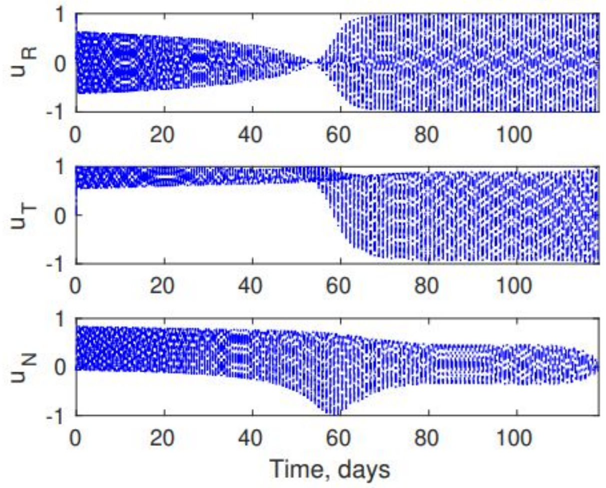
(b) Lyapunov function



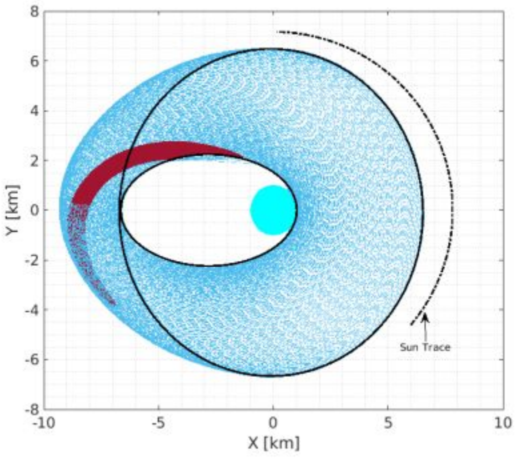
(c) Lyapunov derivative



(d) Control Gains



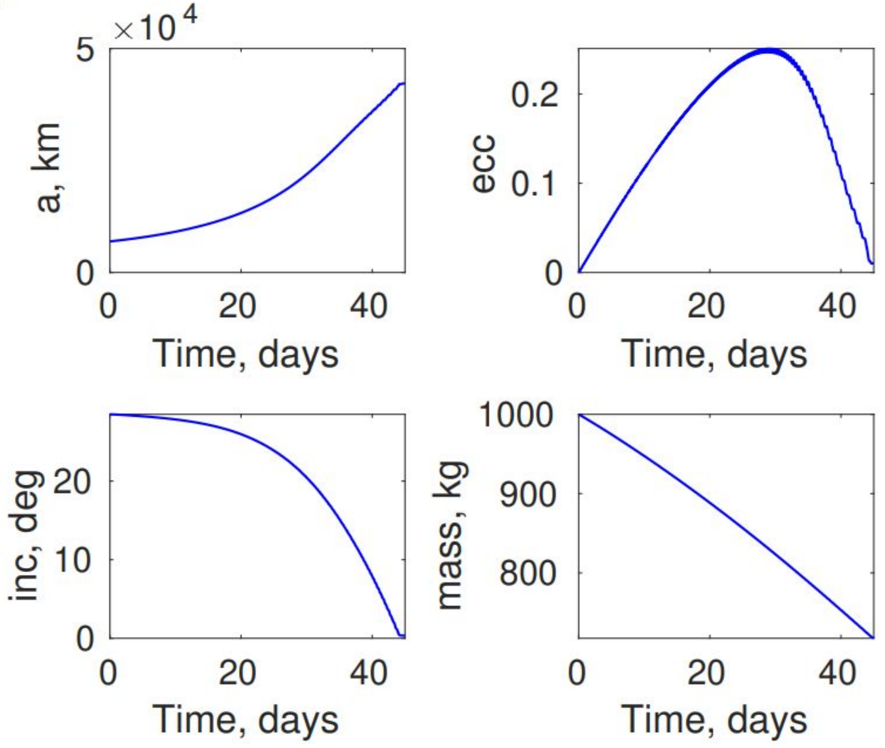
(e) Thrust Vector direction



(f) Orbit Trace

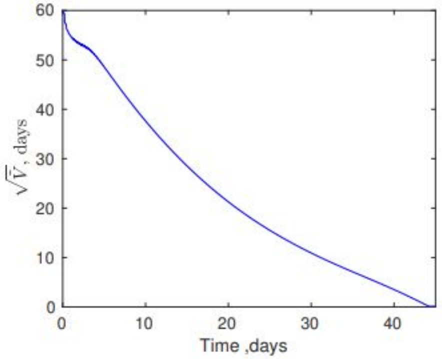
We are pleased to report that Adaptive Q-law performs comparable to literature best offline trajectory planning results while being fully closed-loop and real time implementable

LEO-GEO Transfer – Time Optimal Transfer

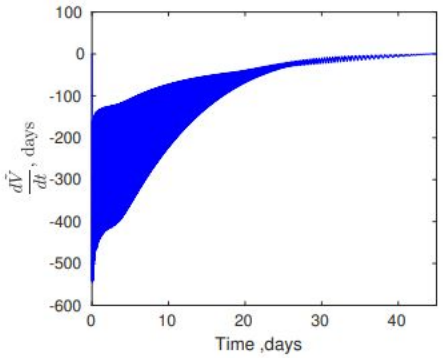


(a) Spacecraft state evolution

Method	Transfer Time, Days
Literature Best Results [Shannon 2020, Leomanni 2021, Graham 2016]	42.3 – 44.48 days
Adaptive Q-law	44.5 days



(b) Lyapunov function



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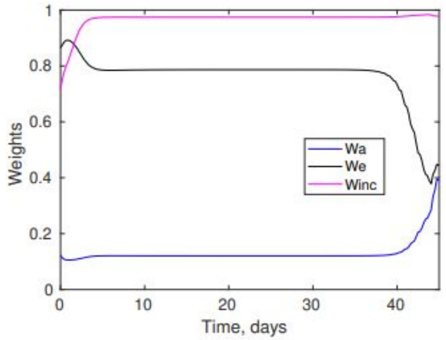

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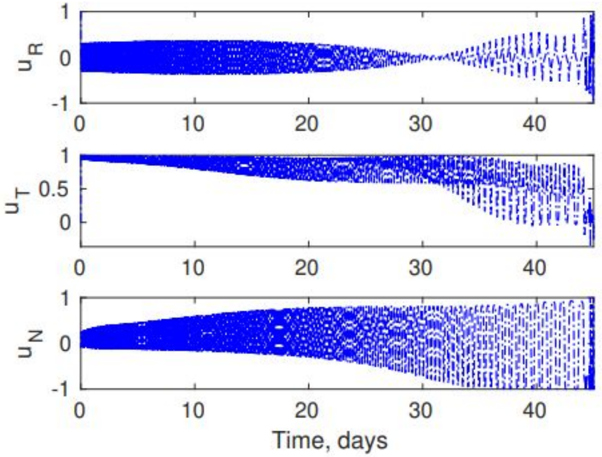
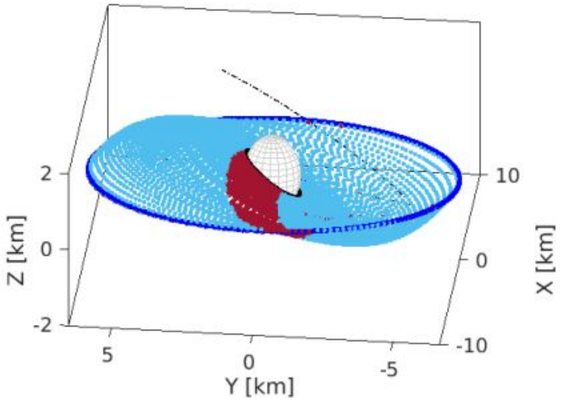


(e) Thrust Vector direction



(f) Orbit Trace

Monte-carlo simulations and Robustness

Error Sources for Simulation

- **Launch Vehicle Injection Error**: zero-mean Gaussian error with 3σ error of 30 km in position and 6 m/s in velocity
- **Navigation Error**: zero-mean Gaussian error with 3σ error of 500 m in position and 50 cm/s in velocity
- **Thrust Vector Error**: Uniform error of 5 deg in azimuth and elevation each drawn from Uniform distribution
- **Environmental Disturbances**: Disturbance torque in spacecraft from solar radiation pressure and gravity gradient

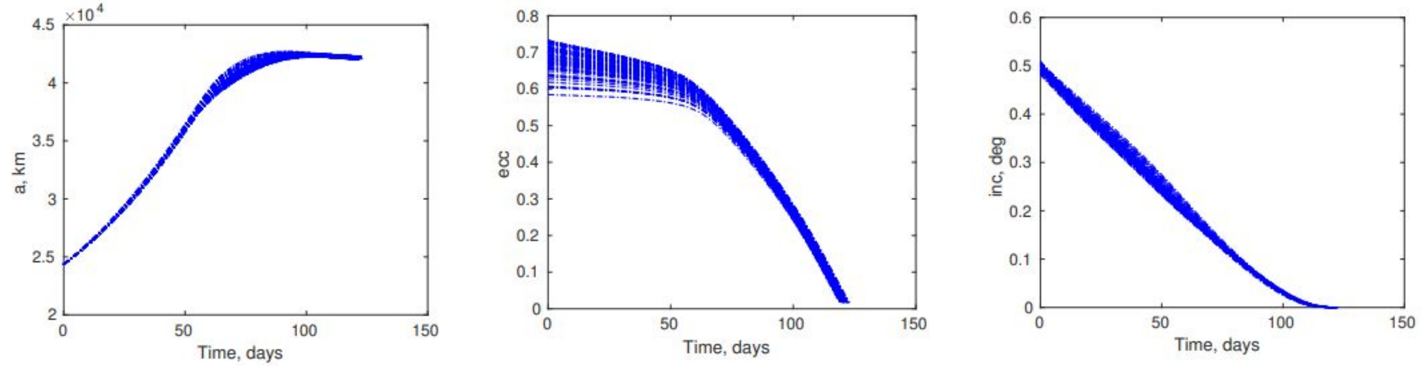


Fig. 8 GTO to GEO Transfer:: Monte-carlo Simulation Results

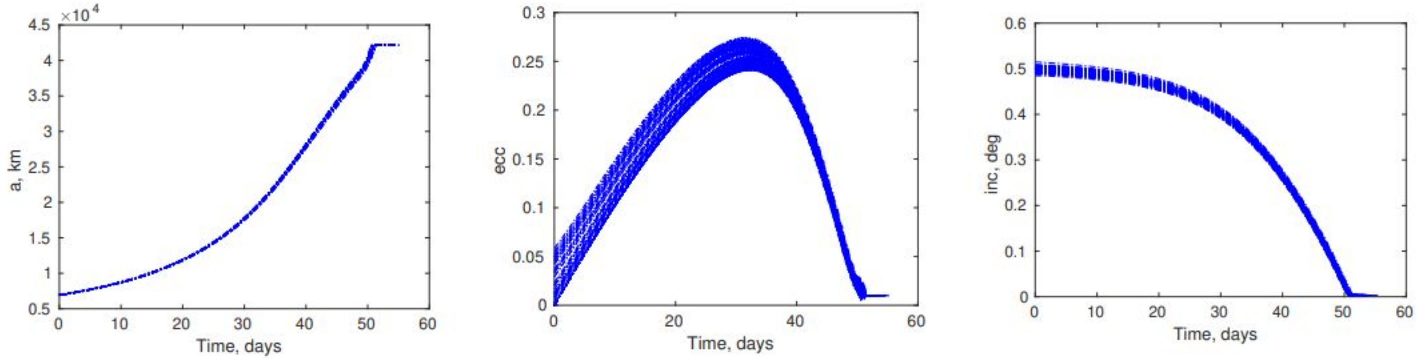


Fig. 9 LEO to GEO Transfer:: Monte-carlo Simulation Results

Conclusion

- We presented a novel adaptive Q-law framework for closed-loop, low-thrust, many-revolution orbit transfers
- Controller gains are learned as a function of state and time using a neural network parameterization, trained through a two-stage, simulation-based optimization strategy with CMA-ES based evolutionary approach
- Simulation studies demonstrate that the proposed algorithm achieves transfer times comparable to those reported in the literature, without any loss in performance.
- This approach enables robust and efficient trajectory planning under practical uncertainties, marking a significant step toward fully autonomous low-thrust guidance.
- In future, we aim to adapt the Neural-AGOS framework to generate time as well as mass optimal trajectory using pareto-optimal studies.

Thank You

Effectivity and coasting arcs

- The proposed formulation along with a gradient free optimizer for gain selection is suitable for near-minimum time transfer problems

- To achieve mass optimal transfers, classical Q-law introduces coasting mechanism to switch off thrusting when its inefficient. We adopt a similar strategy with efficiency directly computed from the modified Lyapunov function rate of change.

$$\dot{\tilde{V}}_{nn} = \min \dot{\tilde{V}}_n$$

$$\dot{\tilde{V}}_{nx} = max\, \dot{\tilde{V}}_n$$

$$\eta_r = \frac{\dot{\tilde{V}}_n - \dot{\tilde{V}}_{nx}}{\dot{\tilde{V}}_{nn} - \dot{\tilde{V}}_{nx}}, \eta_r \in (0,1)$$